

# Characterizing Roundness for Geometric Dimensioning and Analysis

## An Interdisciplinary Guidebook

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### 1. Background

**Metrology, the study of measurement, plays a vital role in nearly all science and engineering processes, from the largest works of civil construction to the very smallest experiments probing the cutting edge of the quantum world.**

While many common dimensional values such as lengths and angles can often be intuitively defined and quantified, others require a more rigorous approach. “Roundness”, or the likeness of a given shape to a perfect circle, is one such value. A framework for quantitative evaluation of the roundness<sup>1</sup> of shapes offers us a valuable tool for use in a myriad of applications where analysis and control of circularity<sup>2</sup> are of interest, such as mechanical design and fabrication, geology and sedimentology, and particle analysis. In this article, we will explore several methods for defining and quantifying roundness, their individual strengths and weaknesses, and some notable applications where such measurements find great utility as part of an ever-improving metrological toolbox.

### 2. Roundness at a Glance

From a general perspective, the *roundness* of a given 2-dimensional shape is a computed value comparing the shape in question to that of a perfect circle. This given shape almost always represents a trace of a real-world object’s perimeter, plotted as data points on a Cartesian plane.

2D traces can be measured using a variety of techniques. Where roundness is a concern for industry and fabrication, traces are often made using contact-based tools and probes. These devices typically rely on a spindle or collet to hold and rotate the workpiece to be

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<sup>1</sup>*Deviation* from roundness also features heavily in such frameworks, as we will see.

<sup>2</sup>*Roundness* is sometimes used alongside the term *circularity* interchangeably. For our purposes, we will use *roundness* to refer to the measured value of a shape’s likeness to a circle, and *circularity* when making general or qualitative statements about shapes’ roundness.

measured, while a finely-calibrated stylus records the radius of the workpiece relative to the spindle over an arbitrary number of even increments. These data are then plotted to form a trace of the workpiece for analysis. Alternatively, in situations where accuracy is less crucial measurement may be obtained using a “3-point method,” in which the workpiece is constrained between two angled blocks and rotated while in contact with a fixed gauge or transducer. Roundness deviation is then taken simply as the average change noted by the gauge [4].

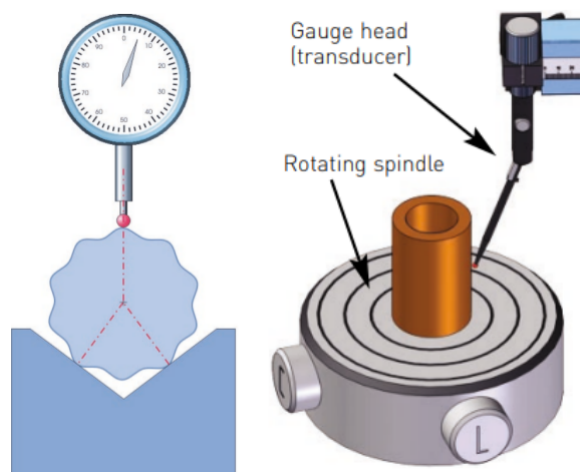


Figure 1: Measurement of roundness via “3-point” method (left) & rotating spindle method (right) [4].

Other measurement methods may also find use in applications where directly tracing an object’s perimeter is impractical, such as the analysis of sediments and other fine particles. In these situations, traces of shapes may instead be obtained from image data using various techniques for image analysis. Such workflows allow shapes captured via imaging to be identified, traced, and examined without any need for physical contact with the specimens.

As methods for producing 2D traces vary wildly by application, so too do the criteria for defining roundness quantitatively — some have become obsolete while others are now ubiquitous, and still others find use in some niche applications.

### 3. Quantitative Definitions

When considering the problem of evaluating a shape’s roundness, one intuitive strategy which may come to mind is to examine the diameter<sup>3</sup> of the shape around the span of its perimeter. As the radius (and thus, diameter) of a perfect circle is constant between any opposing points on its edge, it appears to make sense that a shape’s roundness approaches

<sup>3</sup>While defining *diameter* and *center* is straightforward for perfect circles, it can become tricky when dealing with irregular shapes. In this setting, we may generally define diameter as the distance between two points on the perimeter of a shape separated by 180° relative to the centroid (the average position of all points on the perimeter).

that of a circle as the deviation of its diameter decreases. While enticingly simple, this line of thinking is stymied by the fact that many shapes exist which possess a constant diameter at any point on their perimeter without being especially round whatsoever [1]. To circumvent this issue, a slightly different approach is needed – one which can’t be “fooled” by a shape’s geometry.

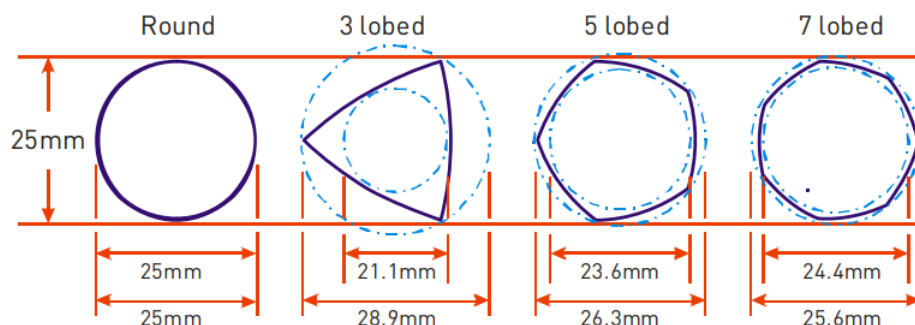


Figure 2: Several shapes with constant diameters, yet varying degrees of circularity [1].

International Organization for Standardization (ISO) standards define roundness as the ratio between the radii of a shape’s largest inscribed circle and smallest circumscribed circle. This method avoids the pitfalls encountered in the aforementioned “constant diameter approach” by considering overall circularity of a shape’s convex and concave points in tandem. According to the ISO model, a perfect circle possesses a roundness value of 1. When measured, all other applicable shapes will give a roundness value between 0-1, with values approaching one as shapes approach a perfect circle [2].

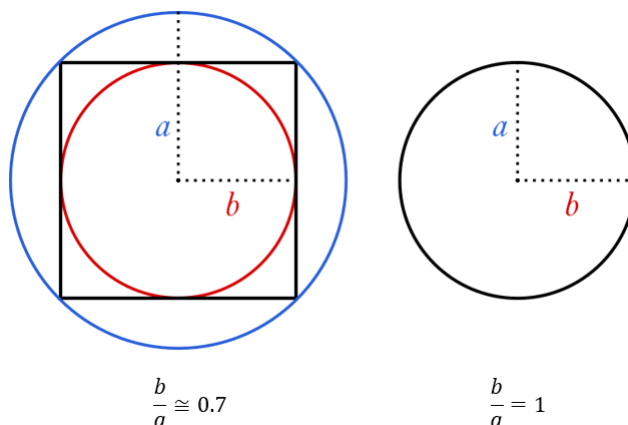


Figure 3: Evaluation of square (left) & perfect circle (right) via ISO roundness model.

Conversely, for many applications it is more practical to describe a shape’s roundness in terms of its deviation from a perfectly round circle (“non-circularity” or “out-of-roundness”). The National Institute of Standards and Technology (NIST) outlines two methods for evaluating roundness deviation from 2D traces: one working from a single trace, and the other using multiple traces in order to minimize measurement error [3].

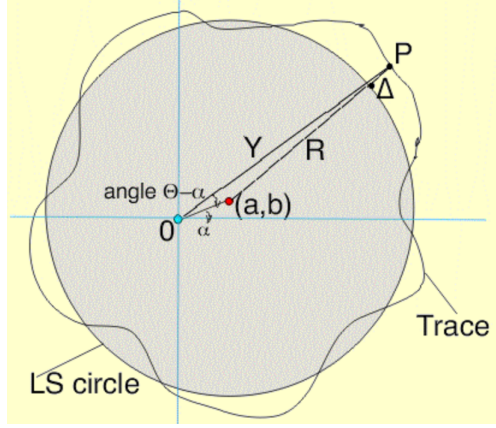


Figure 4: Geometry & Nomenclature used in NIST roundness deviation model [3]. Fitted (“LS”) circle imposed onto initial trace for deviation measurement. Center point (“0”) is arbitrary; denotes center of measurement tool or calculated centroid.

In the “single-trace” method, a trace of the shape is constructed by taking measurements of the radius on points around the edge of the shape at  $N$  evenly-spaced angles  $\theta_i$  for  $i$  on the interval  $[1, N]$ . These data are then subjected to a circular least-squares fit to construct a circle against which the trace can be compared. The radius and centroid of this fitted “reference” circle are given by [3]:

$$R = \frac{1}{N} \sum_{i=1}^N Y_i$$

$$a = \frac{2}{N} \sum_{i=1}^N Y_i \cos(\theta_i)$$

$$b = \frac{2}{N} \sum_{i=1}^N Y_i \sin(\theta_i)$$

where  $R$  is the radius of the fitted circle,  $(a,b)$  is the center of the fitted circle, and  $Y_i$  is the distance between the center of the spindle on a stylus-based measuring instrument and the edge of the trace for angle  $\theta_i$ . Roundness deviation ( $\Delta$ ) for each point on the trace can then be computed and the final  $\Delta$  value is taken as the average of every point’s deviation, computed by:

$$\Delta = Y_i - R - a \cos(\theta_i) - b \sin(\theta_i)$$

The “multi-trace” technique extends the approach described above for use in applications when the roundness of a 3D form (such as a hemisphere) is intended to be used as a standard for roundness. By taking multiple evenly-spaced traces and evaluating the resultant data with a similar least-squares fit, intrinsic roundness error within the instrument used (most commonly a probe or spindle-based device) is separated from that of the 3D form [3].

## 4. Applications

Accurate measurement of roundness forms a crucial part of many metrological processes used in industry, particularly with regards to fabrication. Components manufactured every day around the world rely on strict dimensional tolerancing to properly perform their functions, such as combustion engine cylinders, precision screws, flanges, and countless others. In our increasingly machine-dependent world, the need for versatile, efficient measurement and tolerancing regimens for fabrication and quality control can only be expected to grow. As industry experts and researchers alike strive to ever improve these techniques, a robust toolkit for characterizing roundness remains an integral part of any metrological foundation.

Likewise, roundness characterization plays a valuable role in applications outside the immediate realm of fabrication, a notable example being the field of particle analysis. Though roundness is necessarily a 2D metric for circularity, it may still be applied to characterize 3D particulate objects where it is useful to sample the roundness of particle edges at random orientations. While somewhat niche in scope, this paradigm offers a useful framework for understanding phenomena involving small particulates, such as the weathering of sediments or formation of powdered materials. Such phenomena are relevant to a myriad of endeavors across multiple sectors, from the study of erosion and sedimentation for geospatial information which is vital for conservation and construction projects, to tolerancing and quality control of engineered powders and abrasives.

Methods for obtaining 2D traces in this area often differ from those used for industry and fabrication; small particles are exceedingly difficult to trace mechanically, and thus many processes for roundness characterization in these applications make use of image analysis and segmentation techniques to produce traces for evaluation. This enables small particles to be characterized without the need for physical contact with samples and allows particles to be examined individually as well as in groups. Techniques based on image analysis typically rely on image thresholding and binarization algorithms to isolate particles in an image before tracing their edges. Trace data may then be stored as regions of interest (ROI's) for use in a similar fashion to data procured via physical measurements.



Figure 5: Example of image processing for segmentation & tracing of salt ( $\text{NaCl}$ ) crystals using ImageJ software. Raw image (left) is thresholded to produce binary image (center). Software is then used to produce traces, stored as regions of interest (right).

## 5. Conclusion

While not always as straightforward as other measurement standards, quantitative models for describing the roundness of measured shapes comprise a central component of metrology and play vital roles in countless scientific, engineering, and industrial processes. As our modern world's dependence on technology continues to grow, so too will its demand for precision. Understanding the principles behind the characterization of roundness arms us with an invaluable tool for any metrological arsenal in meeting the challenges of today, as well as those of the future.

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