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An Interview with Professor Piotr Hajłasz

Interviewed by Neil MacLachlan and Lark Song

Biographical Sketch

Piotr Hajłasz was born in Warsaw in 1966. He went to the University of Warsaw for his undergraduate and stayed there until 2004, working up through the ranks to an associate professor after receiving his Ph.D. in 1994 under the supervision of Bogdan Bojarski. As of today, he has been a professor of mathematics at the University of Pittsburgh for a full twenty years. He was elected as a Fellow of the American Mathematical Society in 2017. In recognition of his contribution, he was awarded the prestigious Sierpiński Medal by the Polish Mathematical Society in 2021. Winning this award has placed him among the ranks of distinguished mathematicians including Paul Erdős, Stanisław Ulam, and Benoit Mandelbrot. His research is in geometric function theory which covers a wide range of topics on the borderline of classical analysis, geometric analysis, theory of Sobolev spaces and analysis on metric spaces, where he is known for Hajłasz–Sobolev spaces.

Neil MacLachlan/Lark Song: *Could you describe your early interest in mathematics, how it developed over time, and what motivated you to pursue a career in math?*

Piotr Hajłasz: When I was in the third or fourth grade of elementary school, my father Robert Hajłasz, a well-known math teacher in Warsaw, discovered that I have some talent for mathematics. From that time, he began to teach me intensively. He taught me according to his own plan, which had nothing to do with the school curriculum. He prepared me for math competitions while also showing me the beauty of mathematical abstraction.

I wanted to be a mathematician from the moment I realized how easy and fun mathematics was for me. Learning mathematics and preparing for the Math Olympiad as in 8th grade, younger than most others approaching the competition, made me feel special. I became very competitive – I wanted to be the best among my peers and felt like I had a good chance at achieving that. In retrospect, I am not proud of this kind of mindset, but I was very immature back then. The following year, I began attending a prestigious high school that produced many International Mathematical Olympiad (IMO) winners. There, I realized that there were other students more talented than me, but striving to be the best was very important to me and motivated me to work hard.

I no longer view mathematics in terms of competition. While striving for excellence still motivates me to work hard, I have long ceased wanting to be better than others. My change in attitude occurred during my undergraduate studies. Being competitive is fine, but trying to be better than others is not. Mathematics is not an Olympiad, and in mathematics, there is room for everyone who can make a fruitful contribution. I know my place, and I take pleasure in the successes of others. I now see mathematics through the lens of collaboration instead of competition. I love working with those who are more talented than I am (though it is often very stressful), as well as with those who can learn a lot from me.

I also participated in the International Physics Olympiad, and although I didn't win anything, for a while I wanted to study physics. However, I decided that physics wasn't rigorous enough, and besides, I honestly hated the lab. So, I became a mathematician. I considered doing physics as a second major, but I changed my mind.

[NM/LS]: *How do you approach problem-solving in math, particularly regarding your achievements in Olympiads and other competitions?*

Hajłasz: In Poland, where I grew up, the National Mathematical Olympiad (from which IMO participants are selected) is more popular than in the United States. At least that was the case in my childhood and a significant number of those who became mathematicians had some experience with the Olympiad. In high school, I practiced problems from Olympiad problem sets. The Olympiad brought me a lot of excitement, but I never really liked it and still don't. In high school, I always preferred studying abstract mathematics at the graduate level.

I know that I've shifted the topic and didn't answer the question, but I simply don't know how to answer it. Students often ask how to approach studying for the exams in Calculus. Of course, I give them advice, but honestly, I'm not sure what to say.

[NM/LS]: *How do you think mathematical competitions influence students' interest and future capability in math research?*

Hajłasz: Mathematical Olympiads, as well as other math competitions, allow high school students to discover that they have mathematical abilities. As a result, this may influence their choice of field of study. I am not at all sure whether I would have become a mathematician if not for my father's influence and my successes in the Olympiad.

However, the question of whether Olympiad skills translate to capabilities in math research is much more subtle. I know many IMO winners who have concluded that mathematics is not for them. Olympiad problems are difficult, but with a good idea, they can be solved quickly. On the other hand, math research requires years of study, and often the results come from long and painstaking work. Many steps require ideas at the level of Olympiad problems, but perseverance is key. Furthermore, when working on a problem, there is no certainty that it will be solved. Many IMO winners lack this perseverance and lose their passion for mathematics.

I do want to strongly emphasize, though, that the Math Olympiad is not the only measure of mathematical talent. Many students simply do not find this type of competition suitable. Additionally, math competitions (especially the Putnam Mathematical Competition) require solving problems under time pressure, and speed has nothing to do with mathematical ability. Mathematicians often work on problems for years, so lack of time is not a problem. I have many colleagues who avoided the Olympiad, yet became outstanding mathematicians—often far better than those who achieved great success in the Olympiad. You need some talent to do mathematics, and not everyone has it. However, perseverance is just as important as talent, and I agree with Rockefeller, who said, "I do not think there is any other quality so essential to success of any kind as perseverance."

[NM/LS]: *Who were the significant influences or mentors in your academic journey?*

Hajłasz: My father Robert Hajłasz, my PhD advisor Bogdan Bojarski, Pekka Koskela, and Marek Kordos.

As I wrote earlier, I owe it to my father that I became a mathematician.

Professor Bojarski was the supervisor of both my master's and PhD theses. He wasn't a good lecturer, and apart from suggesting the general topics of my master's and PhD work, he didn't help me at all. Nevertheless, he was an inspiration to me and taught me how to view mathematics. He showed me how to see and understand mathematics without getting into the details of proofs. It's from him that I inherited my geometric intuition and enthusiasm.

In 1994, while working on my doctoral thesis, I spent a month or two in Helsinki. During that time, I visited Jyväskylä for a week at the invitation of Pekka Koskela. It was during this visit that I first realized the mathematics I was creating was appreciated by others, marking the moment I felt I had become a true mathematician. That week was the most intense of my mathematical career. Pekka and I conceived five papers during that time, although it took us ten years to fully write them up. Pekka Koskela was my mentor for many years, and I owe him a great deal.

Marek Kordos was the editor-in-chief of the popular science journal *Delta*. This journal was (and still is) dedicated to the popularization of mathematics, physics, and astronomy. At Marek's suggestion, I worked as an editor for this journal for a few years while I was a PhD student. It's thanks to Marek that I understood the importance of popularizing mathematics, and I learned (or at least I hope so) how to write and talk about it.

[NM/LS]: *Could you explain the significance of your most notable research contributions?*

Hajłasz: My most recognizable mathematical works concern Analysis on Metric Spaces, particularly the theory of Sobolev spaces. I was fortunate enough to be the first to define Sobolev spaces on metric spaces with measure in 1993. Soon after, many other definitions emerged, and the theory developed significantly. It turned out to be important enough that in 2020 it was added to the Mathematics Subject Classification (MSC) as *46E36: Sobolev (and similar kinds of) spaces of functions on metric spaces; analysis on metric spaces.* MSC is a widely-used classification system by the American Mathematical Society that covers all areas of mathematics.

Although my papers on Analysis on Metric Spaces are cited more often than my other papers, I consider my works at the intersection of Analysis, Geometry, and Topology to be deeper. I have contributions in many different areas of Analysis. I simply don't like focusing on one subject for too long or recycling old ideas. I enjoy constantly learning something new and exploring new directions.

[NM/LS]: *What challenges did you encounter in your research, and how did you overcome them?*

Hajłasz: Of course, there are problems I've been working on unsuccessfully for years, but so what? If everything came easily, it wouldn't be fun. When I can't solve one problem, I switch to another. I always have many problems on standby. In my most recent NSF grant application, I listed 21 different open problems, and I'm working on all of them, though not simultaneously.

[NM/LS]: *How did it feel to receive prestigious awards such as the AMS (American Mathematical Society) Fellow, or the Sierpiński Medal?*

Hajłasz: I was very happy–who wouldn't be? Especially after receiving the Sierpiński Medal, as it's the highest honor I've received so far. However, beyond the personal satisfaction of the achievement, I was also pleased that my research area, in which many other mathematicians work, was recognized and appreciated.

[NM/LS]: *Are there any personal anecdotes or stories from your career that you would like to share?*

Hajłasz: I made a bet with a colleague for a case of beer that during a year-long course on differential geometry, I would prepare at least 800 pages of handwritten notes for the students. I ended up writing 917 (the notes are available on my homepage) and won the bet, though my colleague complained that I wrote in letters that were too large. Of course, the bet was not my only motivation for making the notes! Another time, a mathematician asked me a question, offering a bottle of champagne for the solution. However, the problem turned out to be trivial, and within five minutes, I found a counterexample. I didn't get the champagne.

[NM/LS]: *How do you balance your professional and personal life?*

Hajłasz: Ask my wife! She might have a completely different opinion on the matter. When I'm working on a difficult problem, searching for an idea that could lead to a breakthrough, it consumes me entirely. I work 24/7, and nothing else matters. I neglect my family, forget everything, refuse to handle any tasks, and don't reply to most emails. I'm just not available to anyone. The only thing I don't neglect is teaching, as I consider it equally important. I simply can't multitask. Either I'm fully devoted to mathematics, or not at all. These periods can last a week, two, or even longer, and they are very mentally exhausting. My wife understands this perfectly – honestly, I couldn't be luckier. Sometimes, she's just surprised to see me playing a silly video game. When I'm working intensely, I need moments to disconnect and think about nothing, so I might kill zombies, but I can't call the plumber.

Of course, not all of my life is like this. Once I've made a breakthrough and know how to solve a problem (or not, but I got exhausted), writing up the proofs (or not) often doesn't require as much intense focus. Then I have time for refereeing papers, writing recommendation letters, replying to emails (although I always struggle with that), and I'm very present for my family.

Since the work of a mathematician has no fixed hours, I feel like I have no problem balancing work with my personal life, especially with such an understanding wife. Whenever necessary, I can be free, as long as it's not during a period of intense work on a difficult problem.

[NM/LS]: *What do you see as the differences between how math is taught at the K-12 level in the United States as opposed to in other countries?*

Hajłasz: Since I am only familiar with the Polish education system, I will use it for comparison. However, I suspect that education in many other European countries is quite similar. Additionally, I will limit the discussion to education in public high schools. Please keep in mind that I finished school a long time ago, so my knowledge might be slightly outdated (though I've tried to keep up with changes).

In Poland, students who plan to attend university go to high school, while others attend vocational schools. There is no zoning for schools, and acceptance into a high school depends on grades and exam results. Upon entering high school, a student must choose a focus: humanities, biology-chemistry, math-physics, etc. After making this choice, the curriculum is set, and everyone takes the same classes. There is virtually no flexibility as there is in the U.S. Speaking of the math-physics focus, one good thing is that mathematics is offered throughout all four years of high school. Each week, students have many hours of math. In the U.S., math is taught in one-year intensive blocks (Algebra 1, Algebra 2, Precalculus, Calculus), which sometimes results in students having no math at all for a year or more. This is problematic because it doesn't allow students to solidify their knowledge. Furthermore, the math curriculum in Polish high schools doesn't cover as much material as, say, AP Calculus BC, but it focuses more on understanding the material deeply. Although this has likely changed, back in the day, a lot of emphasis was placed on logic and proofs, and every student knew what ε and δ were.

A major difference lies in the qualifications of teachers. In the U.S., the requirements are much lower. A bachelor's degree is required to teach high school, but it doesn't always have to be in mathematics. Sometimes a degree in engineering, physics, or math education (which focuses more on pedagogy than on math itself) suffices. As a result, it's possible for a teacher's education to end at Calculus, and without broader mathematical horizons, it's hard to teach well. As a result, many math teachers don't fully understand what they're teaching, although there are also some exceptional teachers with a lot of knowledge, preparation, and passion. In Poland, a master's degree in mathematics is required, meaning teachers must have taken courses such as Foundations of Mathematics, Linear Algebra, Advanced Calculus, Measure Theory and other math courses too. I personally believe this is the proper standard. However, I don't claim that all Polish teachers are good. Many teach poorly and have very little knowledge, but this is likely the case in many countries.

One thing I like about American schools is the flexibility. My children, while in middle school, were able to take high school classes, and the school helped organize their schedules. Then, in high school, they could take courses at the University of Pittsburgh and receive high school credit for them. Their high school also had a wonderful research program, where students could conduct real research at a university. As a result, many went on to pursue PhDs after finishing college. However, this research program existed only because of one passionate teacher, and the school itself didn't see it as particularly valuable. Such flexibility in Polish schools would be nearly impossible.

What I strongly dislike is that the quality of the school you attend in the U.S. depends entirely on where you live. This leads to racial and economic segregation. In Poland, this kind of segregation doesn't exist.

I can't definitively say which education system is better. I tend to think that for most students, the American system is better, but only if they have access to a good school. As for Polish schools, I believe students who want to study mathematics are better prepared to understand abstract mathematics than they would be in the US.

I have also noticed a difference in the math competitions introduced to younger students, especially in elementary school. In Poland, emphasis is placed on those which require deeper logical thinking. In the U.S., they focus more on speed and quick thinking, at the cost of more creative problem solving.

One of the more interesting competitions is *Math Kangaroo*, open to students in grades K-12. The problems are logical puzzles, so everyone, regardless of knowledge, has a chance to perform well. It is the most popular math competition in the world, with 6,000,000 participants annually. This year, in Poland, Germany, and Brazil, 250,000, 800,000, and 900,000 students participated, respectively. In the U.S., however, despite the competition being held since 1998, only 44,000 participated, simply because schools aren't interested. Let me mention that David Swigon and I organized the *Math Kangaroo* competition here at Pitt from 2007 to 2019, but it was interrupted by COVID.

[NM/LS]: *What do you see as the differences between how math is taught at the undergraduate level in the United States as opposed to Poland?*

Hajłasz: In the US, there is intense competition to get into top universities. In Poland, and generally across Europe, universities are large, public, and free. Most talented students don't face significant challenges getting into them, and in Poland, admission is based solely on the results of the matura exams, which are equivalent to the SAT (perhaps combined with high school grades, though I'm not entirely sure). As a result, everyone starts on more equal footing, as they're all attending the same universities.

In Poland, when you begin your studies, you must immediately choose a major. This means that if you choose mathematics, you'll be studying mathematics, mathematics, and more mathematics, along with some computer science and maybe physics. Of course, general education courses are also required, but to a much lesser extent than in the US. There is also the possibility of combining mathematics with other fields of study (similar to a major and minor system), but even then, you need to know from the beginning what you want to study.

In the US, you start university without choosing a specific major, and only after some time do you decide on a major. If it doesn't suit you, you can change it.

Which system is better? For most students, the American system is definitely better because, after high school, very few people truly know what they want to study. However, for students who have been determined to study mathematics since high school, the Polish system is better.

At the University of Warsaw, studies are offered in a 3+2 system: three years for a bachelor's degree and two for a master's. Each year, 240 math majors are admitted, but nearly half drop out after the first year, and those students must reapply to the university from the beginning! Around 100 students graduate with a bachelor's in math annually, and 60 of them go on to earn a master's degree – all of it free of charge! It's a brutal selection process, but after obtaining a master's degree, students are truly well-prepared for a PhD in mathematics. In the US, because students don't have to decide on a major right away and are required to take many general education courses, the requirements for earning a bachelor's degree are minimal. Many students apply to PhD programs in mathematics right after obtaining their bachelor's degree, as master's programs are very expensive, and as a result, they are drastically underprepared for a PhD program relative to international counterparts.

[NM/LS]: *What aspects of how math is taught in Poland do you think would be beneficial if incorporated into pedagogy in the United States?*

Hajłasz: I will answer this question in the context of Pitt because I know the program at Pitt well. However, I assume that my answer will apply to many American universities. This is not an easy question because the University of Warsaw is the best Polish university, attracting many Math Olympiad winners every year. At Pitt, although we have very talented math majors each year, only a small handful are interested in pursuing a PhD in mathematics.

The courses offered at the University of Warsaw are very advanced, and freshmen are required to take subjects like Foundations of Mathematics (with cardinal and ordinal numbers and a rigorous proof of the well-ordering theorem and Zorn's Lemma), Advanced Calculus I (one variable for the entire year), Linear Algebra (similar to our graduate-level course), and probably something more. Sophomores learn Advanced Calculus II with measure theory and integration of differential forms on manifolds and many other courses. No wonder half the students drop out after the first year. All courses are taught rigorously, with a strong emphasis on proofs. Such a program at Pitt would make no sense whatsoever, and for some students at the University of Warsaw, it probably doesn't either. However, for those who want to pursue a PhD, it is a very good program.

In my opinion, the problem with mathematical education at Pitt is that the requirements for obtaining a major in math are minimal, and for talented students who would like to learn more, there is no clear path regarding which courses they should take and in what order. I believe that something like an 'Advanced Track to Mathematics' should be created, a program that would outline a more advanced pathway. I know something like this exists at Michigan State University, but I am not familiar with the details. I would like to work on creating something similar at Pitt, provided that others support me in this initiative. However, it's not that simple. For instance, students should have the opportunity to determine whether such an accelerated path would be a good choice for them. How can this be done? Additionally, courses, or at least some of them, offered in our graduate program should have a syllabus that assumes only the material students could have mastered in our undergraduate program.

[NM/LS]: *Could you briefly describe the graduate curriculum at Pitt? What aspects of this do you think should be changed and how should they be changed?*

Hajłasz: I'm talking too much already, so I think I'll wrap it up and answer the question with a short response. I really like our graduate program. I know students complain about our Preliminary Exam, but I think it's necessary, and I wouldn't change anything about it. As for the graduate-level courses offered to our students, I don't see the possibility (or the need) to increase their number because students have to teach a lot (in Europe, they barely do!), and if they took more courses, they wouldn't have time for research. I'm a bit concerned that many students delay choosing a PhD advisor for too long. This greatly delays the start of their research work.

This interview began over the summer and concluded on September 15, 2024, with Professor Hajłasz contributing supplementary refinements during his global travels. The process was enjoyable thanks to his sharp wit and sense of humor!

— Neil MacLachlan and Lark Song

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