Proofs Without Words

Series 4

Paul Gartside

(Communicated by Stephan Mirtchev)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

area of parallelogram
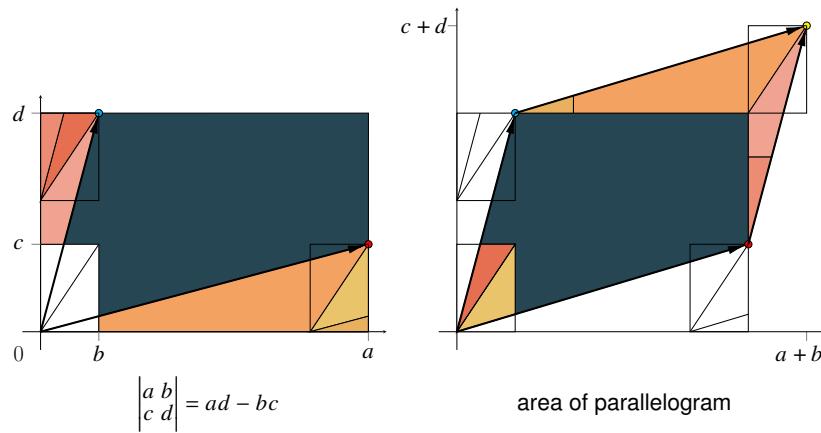
A $2{\times}2$ Determinant is the Area of a Parallelogram.

A triple (a, b, c) of natural numbers is *Pythagorean* if $a^2 + b^2 = c^2$.

Theorem. There is a bijection between Pythagorean triples (a, b, c) and factorizations of even squares, $n^2 = 2pq$.

$c^2 = a^2 + b^2 - n^2 + 2pq$
$\therefore \qquad c^2 = a^2 + b^2 \iff n^2 = 2pq \qquad \square.$

Corollary. There are infinitely many Pythagorean triples.

Enumerating the Pythagorean triples.
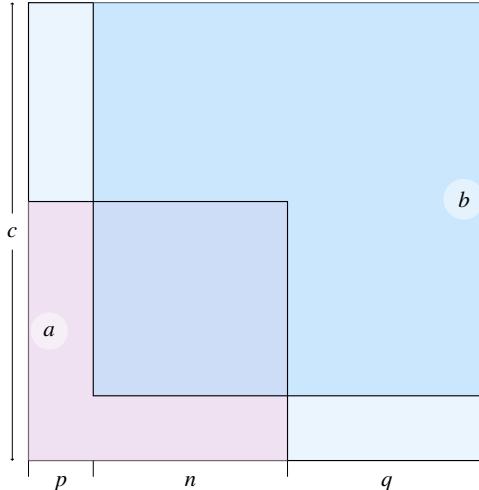
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